Hong Kong Mathematics Olympiad (2015/2016)

Final Event 1 (Group)

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score				Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特别声明，答案须用数字表达，并化至最简。

1. 一项工程包括三个项目：*A*，*B* 和 *C*。若项目 *A* 开始三天后，项目 *B* 才可开始进行。项目 *C* 亦必须在项目 *B* 开始四天后才可开始进行。若完成项目 *A*，*B* 和 *C* 分别需要四天，六天和五天，求最少天数(P)完成全项工程。

A project comprises of three tasks, *A*, *B* and *C*. Suppose task *B* must begin 3 days later than task *A* begins, and task *C* must begin 4 days later than task *B* begins. If the numbers of days to complete tasks *A*, *B* and *C* are 4, 6 and 5, respectively, determine the least number of days (P) to complete the project.

P =

2. 指示牌上挂有红、黄、绿闪灯。红、黄、绿闪灯分别每隔 3 秒、4 秒、8 秒闪烁一次。当 0 秒时，红、黄、绿闪灯同时闪烁。若当 Q 秒时，第三次出现只有红及黄闪灯同时闪烁，求 Q 的值。

There are 3 blinking lights, red, yellow and green, on a panel. Red, yellow and green lights blink at every 3, 4 and 8 seconds, respectively. Suppose each light blinks at the time $t = 0$. At time Q (in seconds), there is the third time at which only red and yellow lights blink, determine the value of Q.

Q =

设

3.
$$f_{n+1}=\begin{cases}f_n+3 & \text{若 } n \text{ 是双数}\\ f_n-2 & \text{若 } n \text{ 是单数}\end{cases}。$$

若 $f_1=60$，求 n 的最少可能值，令当 $m\geq n$ 时，满足 $f_m\geq 63$。

Let
$$f_{n+1}=\begin{cases}f_n+3 & \text{if } n \text{ is } even\\ f_n-2 & \text{if } n \text{ is } odd\end{cases}.$$
If $f_1=60$, determine the smallest possible value of n satisfying $f_m\geq 63$ for all $m\geq n$.

n =

4. 求 $T=\left(3^{2^0}+1\right)\times\left(3^{2^1}+1\right)\times\left(3^{2^2}+1\right)\times\cdots\times\left(3^{2^{10}}+1\right)$ 的值。

Determine the value of $T=\left(3^{2^0}+1\right)\times\left(3^{2^1}+1\right)\times\left(3^{2^2}+1\right)\times\cdots\times\left(3^{2^{10}}+1\right)$.

T=

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Final Event 2 (Group)

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1. 一个盒子有五个球，球面上分别印上号码 3、4、6、9 或 10。由盒中同时随机取出 2 个球，并得出其号码的总和。若 A 为不同总和的数量，求 A 的值。

A box contains five distinctly marked balls with number markings being 3, 4, 6, 9 or 10. Two balls are randomly drawn without replacement from the box. If A is the number of possible distinct sums of the selected numbers, determine the value of A.

A =

2. 设 $f_1 = 9$ 及

$$f_n = \begin{cases} f_{n-1} + 3 & \text{若 } n \text{ 是 3 的倍数} \\ f_{n-1} - 1 & \text{若 } n \text{ 不是 3 的倍数} \end{cases}。$$

若 B 为 k 的值的可能数量，其中 k 满足 $f_k < 11$，求 B 的值。

Let $f_1 = 9$ and

$$f_n = \begin{cases} f_{n-1} + 3 & \text{if } n \text{ is a multiple of } 3 \\ f_{n-1} - 1 & \text{if } n \text{ is not a multiple of } 3 \end{cases}.$$

If B is the number of possible values of k such that $f_k < 11$, determine the value of B.

B =

3. 设 $a_1, a_2, a_3, a_4, a_5, a_6$ 为非负整数，并满足

$$\begin{cases} a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 + 6a_6 = 26 \\ a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 5 \end{cases}。$$

若 c 为方程系统的解的数量，求 c 的值。

Let $a_1, a_2, a_3, a_4, a_5, a_6$ be non-negative integers and satisfy

$$\begin{cases} a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 + 6a_6 = 26 \\ a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 5 \end{cases}.$$

If c is the number of solutions to the system of equations, determine the value of c.

c =

4. 设 d 及 f 为正整数及 $a_1 = 0.9$。若 $a_{i+1} = a_i^2$ 及

$$\prod_{i=1}^{4} a_i = \frac{3^d}{f},$$

求 d 的最少可能值。

Let d and f be positive integers and $a_1 = 0.9$. If $a_{i+1} = a_i^2$ and

$$\prod_{i=1}^{4} a_i = \frac{3^d}{f},$$

determine the smallest possible value of d.

d =

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Final Event 3 (Group)

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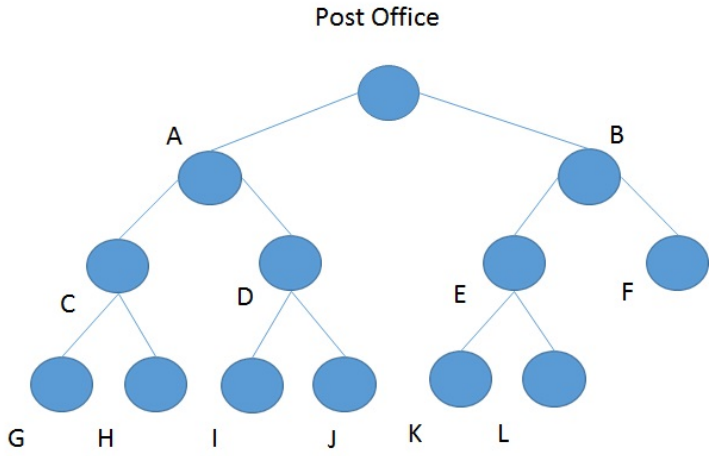
1. 下图是邮差的送信路线图：从邮局开始，到达十二个地点送信，最后返回邮局。若邮差从一地点步行到另一相连地点需要十分钟及 K 为邮差需要的时数来完成整天路线，求 K 的最少可能值。

The figure below represents routes of a postman. Starting at the post office, the postman walks through all the 12 points and finally returns to the post office. If he takes 10 minutes from a point to another adjacent point by walk and K is the number of hours required for the postman to finish the routes, find the smallest possible value of K.



K =

2. 若 n 为正整数，$a_1 = 0.8$ 及 $a_{n+1} = a_n^2$，求 L 的最少值，满足
$$a_1 \times a_2 \times \cdots \times a_L < 0.3$$。

If $a_1 = 0.8$ and $a_{n+1} = a_n^2$ for positive integers n, determine the least value of L satisfying
$$a_1 \times a_2 \times \cdots \times a_L < 0.3.$$

L =

3. 若方程 $\sqrt[3]{5+\sqrt{x}}+\sqrt[3]{5-\sqrt{x}}=1$，求实数根 x。

Solve $\sqrt[3]{5+\sqrt{x}}+\sqrt[3]{5-\sqrt{x}}=1$ for real number x.

x =

4. 若 a, b 及 y 为实数，并满足

$$\begin{cases} a+b+y=5 \\ ab+by+ay=3 \end{cases},$$

求 y 的最大值。

If a, b and y are real numbers and satisfy

$$\begin{cases} a+b+y=5 \\ ab+by+ay=3 \end{cases},$$

determine the greatest possible value of y.

y =

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Final Event 4 (Group)

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1. 若 a^2 及 b^2 为整数，且相差 144，求 $d = a + b$ 的最大值。

   If a^2 and b^2 are two integers that differ by 144, determine the largest possible value of $d = a + b$.

d =

2. 若 n 为整数，n^2 的个位数及十位数分别为 u 及 7，求 u 的值。

   If n is an integer, and the unit and the tens digits of n^2 are u and 7, respectively, determine the value of u.

u =

3. 求实数

$$c=\frac{\left(4+\sqrt{15}\right)^{\frac{3}{2}}+\left(4-\sqrt{15}\right)^{\frac{3}{2}}}{\left(6+\sqrt{35}\right)^{\frac{3}{2}}-\left(6-\sqrt{35}\right)^{\frac{3}{2}}}$$

的值。

Determine the value of real number

$$c=\frac{\left(4+\sqrt{15}\right)^{\frac{3}{2}}+\left(4-\sqrt{15}\right)^{\frac{3}{2}}}{\left(6+\sqrt{35}\right)^{\frac{3}{2}}-\left(6-\sqrt{35}\right)^{\frac{3}{2}}}.$$

c =

4. 求实数

$$x=1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{x}}}}$$

的正数值。

Determine the positive value of the real number

$$x=1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{x}}}}.$$

x =